Investigating Surface Area and Volume of Containers

I. ASSESSMENT TASK OVERVIEW & PURPOSE:
My task will illustrate a student's knowledge and understanding of Surface Area compared to Volume. Students will also explore 2-dimensional and 3-dimensional figures through nets.

II. UNIT AUTHOR:
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III. COURSE:
Geometry

IV. CONTENT STRAND:
Geometry, Measurement

V. OBJECTIVES:
Students should be able to use surface area and volume formulas to explore a real life scenario.

VI. REFERENCE/RESOURCE MATERIALS:
Student/Teacher Handout Attached
Conversion for cubic centimeters to fluid ounces

VII. PRIMARY ASSESSMENT STRATEGIES:
Students will complete the worksheet and the teacher should assess understanding throughout. At the conclusion of this activity, students should individually answer the reflection questions.

VIII. EVALUATION CRITERIA:
Rubrics attached

IX. INSTRUCTIONAL TIME:
90 minutes

Lemonade Business

Strand
Geometry, Measurement

Mathematical Objective(s)
Students will use their knowledge of 2D and 3D shapes to "construct" containers for their lemonade stand. Students will also have use conversions to find the correct size containers. Student will also be expected to use nets to create their 3D container based on their drawing.

Related SOL

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

NCTM Standards

- analyze properties and determine attributes of two- and three-dimensional objects
- use geometric models to gain insights into, and answer questions in, other areas of mathematics

Materials/Resources
Student/Teacher Handout attached
Calculators
Scissors
Compass
Construction Paper
Geometry Formula Sheet to help with the necessary formulas
Conversion for cubic centimeters to fluid ounces

Assumption of Prior Knowledge
Students should know the formulas to find surface areas and volumes. Students should be at Van Hiele Level 2 Analysis when beginning this activity meaning they should recognize different 3D shapes and be able to draw different types. (Students will struggle to find dimensions of a container that will hold the 12 ounces and in addition to this will have to know the correct time to utilize surface area compared to volume.) As a class, students should have already discussed volume as a measurement of what an object can hold, while surface area measures the material necessary to construct an object but students will have to be able to do both of these when phrased differently. During this activity, you could discuss the size of soda cans and soups cans and discuss why they are the size they are and whether or not that

is a size that would minimize cost. That is included on the teacher worksheet. As a part of the discussion, have students share the dimensions and cost of their containers and build a model.

Introduction: Setting Up the Mathematical Task

In this task/activity, you will investigate the relationship between surface area and volume. You will also build a 3D model of your container from a 2D net in order to hold a given volume. Your hope is to minimize materials used so you can earn more money in the end.

Time: This activity will take 45 – 90 minutes plus additional time for discussion and reflection

Introduction: Have students work through the student worksheet. While students are completing their worksheets, they will create a model for the container they designed. Students can then present their container with the cost it would be to make their container. Teacher should display the containers with the cost around the room so students can see. The worksheet should help guide the student thinking but the teacher should remind students of the formulas they know for the volume and surface area of the different shapes.

This activity should really begin as an exploration with very little teacher input. Students should feel free to make any size container they want. After students create their containers, students should be given time to form conjectures about the size of the container that would minimize cost.

The teacher should then lead a discussion with the class which allows students to explain their thinking and to reflect on the activity as a whole.

Student Worksheet Name : _____

You decide that you make the best lemonade in town and it is time to start selling it. You make 240 fluid ounces that you plan to sell by distributing into 20 different containers.

a) How many ounces must each container hold??

b) Draw a sketch of what you want your shape to look like. On a separate sheet of paper, draw a net of your container. Then construct the container you designed.

c) Determine the dimensions of your shape and find the volume. Remember it must hold the correct number of ounces from part a. Your can must hold at least 12 ounces but not over 13 ounces.

STOP: After you have your dimensions, ask your teacher to verify the dimensions that you are using as accurate. If they are not, you are going to have to go back to part b and try again ☺

d) If materials for the container cost \$0.04 for every 100 cm^3. How much will the materials for your container cost? **Write the cost to create your container on the figure so it can be displayed around the classroom.**

e) You plan on selling each container of lemonade for \$0.99. How much profit will you make? Assume your parents are providing the materials for making the lemonade (at least initially ☺) so you only have to cover the cost of the material for the containers.

Reflection:

1) Look around the classroom. In your class, which container is the cheapest to produce? What are the dimensions of that container?

2) Investigate: Do you think you could create a container that would be even cheaper to produce than the one from your class? What changes would you make?

3) What is the relationship (if one exists) between surface area and cost? Volume and cost?

4) Find the dimensions of a soda can and measure the surface area. Is a soda can the cheapest type of cylinder a company could produce?

Student Exploration

This activity can be completed individually, pairs or in a small group.

Whole Class Sharing/Discussion (if relevant): At the conclusion of the lesson, students should be given time to reflect before a whole group discussion.

Student/Teacher Actions:

- Students will complete the worksheet while the teacher facilitates the work. Teachers should watch closely for mistakes in the early sections which will lead to misconceptions in the later part of the worksheet. Attached is a sheet of acceptable size containers.
- Teachers should monitor progress and check the sketches.
- Teacher should be ready to help students understand the connection between surface area and cost. Also making sure to that students are correctly using the units and the cost since it is for 100 cm^3

Monitoring Student Responses

Students should work with a partner to help guide them through the worksheet. The teacher will get a great idea of how students understand the material by the creation of their containers. Teachers should utilize the excel spreadsheet to check the progress of individual students or partners.

Assessment List and Benchmarks

- Included is a key which should help the teacher guide students through the work.
  The excel file attached would help a teacher if the students chose a cylinder. I would recommend creating similar files for cones, pyramids and prisms.

- Student Worksheet and Reflection
- Sample Rubrics
  - http://www.rubrics4teachers.com/pdf/PerformanceTaskRubric.pdf

Benchmarks:

As your students complete this activity, keep in mind that it is very student/partner oriented. Many of the answers will vary so, as the teacher, be ready to accept many different answers. An excel sheet is available with the document so you can quickly check students' answers. This activity shows students an introductory example of problems they will encounter in higher levels of math.

a) How many ounces must each container hold?? **12 ounces**

b) Draw a sketch of what you want your shape to look like. Consider how you want the radius of your container to compare to the height.

ANSWERS WILL VARY

c) Determine the dimensions of your shape. Remember it must hold the correct number of ounces from part a.

Provide students with the conversion: $1\ cm^3 \approx .0338\ oz^3$ or use this website:

http://www.unitconversion.org/volume/cubic-centimeters-to-fluid-ounces-us-conversion.html

Check for accuracy using the excel sheet which will provide you with sample measurements.

d) If materials for the container cost \$0.04 for every 100 cm^3. How much will the materials for your cylinder cost?

Check for accuracy using the excel sheet which will provide you with sample measurements.

e) You plan on selling each container of lemonade for \$0.99. How much profit will you make? Assume your parents are providing the materials for making the lemonade (at least initially ☺) so you only have to cover the cost of the material for the containers.

Check for accuracy using the excel sheet which will provide you with sample measurements.

Reflection:

1) Look around the classroom. In your class, which container is the cheapest to produce? What are the dimensions of that cylinder? **ANSWERS WILL VARY**

2) Investigate: Do you think you could create a container that would be even cheaper to produce than the one from your class? What changes would you make?

3) What is the relationship (if one exists) between surface area and cost? Volume and cost?

Students should recognize that it is the surface area measurement which changes the cost. All of the volumes should be relatively equal whereas the changing surface areas is the measurement which will affect the cost.

4) Find the dimensions of a soda can and measure the surface area. Is a soda can the cheapest type of cylinder a company could produce?

A soda can is approximately 3.2 cm diameter and 12.2 cm height. By looking at the excel sheet you can see that the lowest surface area occurs with a radius of approximately 4 cm and a height of 7 cm. Have students use this opportunity to maybe discuss why cans are not made that size.

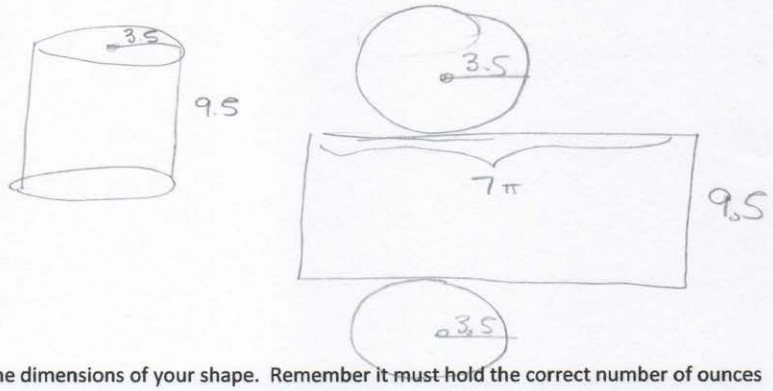
Student Worksheet **Name :** Student A

You decide that you make the best lemonade in town and it is time to start selling it. You make 240 fluid ounces that you plan to sell by distributing into 20 different containers.

a) How many ounces must each container hold??

$\frac{240}{20} = 12$ ounces

b) Draw a sketch of what you want your shape to look like. On a separate sheet of paper, draw a net of your container. Then construct the container you designed.



c) Determine the dimensions of your shape. Remember it must hold the correct number of ounces from part a. Your can must hold at least 12 ounces but not over 13 ounces.

$r = 3.5$
$h = 9.5$

$V = 365.6\ cm^3$
≈ 12.3 ounces ✓

STOP: After you have your dimensions, ask your teacher to verify the dimensions that you are using as accurate. If they are not, you are going to have to go back to part b and try again ☺

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d) If materials for the container cost \$0.04 for every 100 cm^3. How much will the materials for your container cost? $SA = 2\pi r^2 + 2\pi rh$

$2\pi(3.5)^2 + 2\pi(3.5)(9.5) = 285.9$

$C = \frac{285.9}{100} \cdot \$.04 = \$.114 / \text{container}$

e) You plan on selling each container of lemonade for \$0.99. How much profit will you make? Assume your parents are providing the materials for making the lemonade (at least initially ☺) so you only have to cover the cost of the material for the containers.

$.99 - .114 = \$.876$

f) Write the cost to create your container on the figure so it can be displayed around the classroom.

Reflection:

1) Look around the classroom. In your class, which container is the cheapest to produce? What are the dimensions of that container?

2) Investigate: Do you think you could create a container that would be even cheaper to produce than the one from your class? What changes would you make?

3) What is the relationship (if one exists) between surface area and cost? Volume and cost?

4) Find the dimensions of a soda can and measure the surface area. Is a soda can the cheapest type of cylinder a company could produce?